Gauging noneffective group actions & mirror symmetry

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(work in progress w/ Tony Pantev)

Today I'll describe some work on 2D theories called "gauged sigma models."

Recall:

- "Sigma model on X" is a (2D) theory of maps
  ϕ : (2D worldsheet) $\longrightarrow X$

  $$S = \int d^2x \left[g_{i\bar{\jmath}} \, \partial\phi^i \, \bar{\partial}\phi^{\bar{\jmath}} + \cdots \right]$$

- If X admits the action of a group G, then can gauge the action of G on the sigma model above.
  - $\rightarrow$ have G-gauge field on worldsheet
  - $\rightarrow$ replace $\partial\phi^i$ with $D\phi^i$

  etc

- If G is finite,
  a G-gauged sigma model = "orbifold"

Let X be a manifold w/ an action of a group G.
Consider a G-gauged sigma model on X.

For G to act "noneffectively" means some of the elements of G act trivially.
Such elements form a normal subgroup, call it K.

1^{st} question:

is the G-gauged sigma model,

the same as

a (G/K)-gauged sigma model ?

The answer is <u>no</u>, as we'll see next.

Ex Let X be a manifold w/ a $\mathbb{Z}_2 \times \mathbb{Z}_2$ action.
The dihedral group D_4 obeys

$$1 \to \mathbb{Z}_2 \to D_4 \to \mathbb{Z}_2 \times \mathbb{Z}_2 \to 1$$

so we can define a D_4-orbifold of X by,
for any $g \in D_4$, g acts by 1st mapping to $\in \mathbb{Z}_2 \times \mathbb{Z}_2$.

Compare $Z_{1\text{-loop}}(D_4)$ to $Z_{1\text{-loop}}(\mathbb{Z}_2 \times \mathbb{Z}_2)$

$$Z_{1\text{-loop}} \sim (\text{sum over } D_4 \text{ bundles})(Z_{\text{bundle}})$$

$$= \frac{1}{|D_4|} \sum_{\substack{g,h \in D_4 \\ gh = hg}} Z_{g,h}$$

Let $\bar{g}$ denote image of $g \in D_4$ in $\mathbb{Z}_2 \times \mathbb{Z}_2$. (so, $Z_{g,h} = Z_{\bar{g},\bar{h}}$)

Write $D_4 \equiv \{1, a, b, z, az, bz, ab, ba = abz\}$
$\mathbb{Z}_2 \times \mathbb{Z}_2 \equiv \{1, \bar{a}, \bar{b}, \bar{a}\bar{b}\}$

then for $\bar{b}\,\underset{\bar{a}}{\square}$ in $Z(\mathbb{Z}_2 \times \mathbb{Z}_2)$, <u>no</u> corresponding sector in $Z(D_4)$.

$\therefore Z_{1\text{-loop}}(D_4) = \# \Big[Z_{1\text{-loop}}(\mathbb{Z}_2 \times \mathbb{Z}_2) - (\text{an } SL(2,\mathbb{Z}) \text{ orbit of twisted sectors}) \Big]$

$\therefore Z_{1\text{-loop}}(D_4) \neq Z_{1\text{-loop}}(\mathbb{Z}_2 \times \mathbb{Z}_2)$

$\therefore D_4$ gauging $\neq$ $\mathbb{Z}_2 \times \mathbb{Z}_2$ gauging

Massless spectrum:

For G finite, massless spectrum $= \bigoplus_{[g]} H^*(X^g)^{Z(g)}$

- known for G effectively-acting
- also true for G noneffectively-acting

Ex $[X/\mathbb{Z}_k]$ where all of $\mathbb{Z}_k$ acts trivially

According to claim above,
massless spectrum $=$ k copies of $H^*_{DR}(X)$

Note: $Z_{1\text{-loop}}(\mathbb{Z}_k) = \frac{1}{|\mathbb{Z}_k|} \sum_{g,h} Z_{g,h} = \frac{k^2}{k} Z(X) = k\, Z_{1\text{-loop}}(X)$

Ordinarily in QFT, ignore factors in front of partition f'ns, but can not in a theory coupled to (worldsheet) gravity.

In particular, in worldsheet string theory such factors contain information on state degeneracies.

After all, $Z_{1\text{-loop}} \sim (\#) \int_F (\#) \sum_{\mathcal{H}} q^{L_0} \bar{q}^{\bar{L}_0}$

so multiplying $Z_{1\text{-loop}}$ by k $\sim$ increasing number of states at each $(L_0, \bar{L}_0)$ by factor of k

Consistent ✓

Deformations

Ex $[X/\mathbb{Z}_k]$ where all of $\mathbb{Z}_k$ acts trivially

Massless spectrum is k copies of $H^*(X)$
so there are k times as many physical moduli
as moduli of X.

Untwisted sector clear - just deform X.
But the other $(k-1)$ sectors?

Deforming along these directions leads to
some (new) abstract CFT's,
which we'll be able to ~~under~~ see & understand
explicitly.

To begin to understand those deformations, first notice:

twist field for trivially-acting $\mathbb{Z}_k$ generator

$$= \text{field valued in } k^{th} \text{ roots of unity}$$

Counting matches: exactly k possible powers in both cases

Correlation f'ns match:

Quantum symmetry in orbifold says

$$\langle \Upsilon^n \cdots \rangle = 0 \quad \text{unless} \quad k | n$$

If Υ is field valued in k^{th} roots of unity, then

$$\langle \Upsilon^n \cdots \rangle = \int [D \cdots] \sum_{\Upsilon} \Upsilon^n \cdots$$

$$= 0 \quad \text{unless} \quad k | n$$

Deformations, cont'd

Thus, for example, if we orbifold a LG model by a trivially-acting $\mathbb{Z}_k$,
then deformations $\rightsquigarrow$ superpotentials of form

e.g. $$W = x_1^5 + \cdots + x_5^5 + \underline{Y}\, x_1 x_2 x_3 x_4 x_5$$

where $x_1, \cdots x_5$ are ordinary chiral superfields
$\underline{Y}$ is field valued in k^{th} roots of unity

We shall see that this same structure emerges from completely different considerations when studying mirror symmetry.

A non-effective, non-finite quotient

Consider a 2D (2,2) $U(1)$ gauge theory, defined by N chiral superfields, each of charge k w.r.t. $U(1)$

$$\sim \; (\mathbb{C}^N - 0)/\mathbb{C}^\times$$

Perturbatively, identical to the $\mathbb{CP}^{N-1}$ model, but nonperturbatively different.

Ex $\mathbb{CP}^{N-1}$ $\quad U(1)_A \longmapsto \mathbb{Z}_{2N}$ by instantons

Here, $\quad U(1)_A \longmapsto \mathbb{Z}_{2kN}$ " "

Ex Quantum cohomology of $\mathbb{CP}^{N-1}$: $\mathbb{C}[x]/(x^N - q)$

" " here : $\mathbb{C}[x]/(x^{kN} - q)$

Different physics

Morally, this is a local orbifold by a triv'ly acting $\mathbb{Z}_k$. We've already seen that gauging trivially-acting groups gives nontrivial results, so should not be surprised to see analogous phenomena here.

Mirror symmetry a la Hori-Vafa-Morrison-Plesser:

1^{st} build intermediate superpotential

$$W_{int} = \underset{\substack{\downarrow \\ \text{gauge multiplet}}}{\Sigma}\left(\sum_i Q_i Y_i \right) + \sum_i e^{-Y_i}$$

$$= \Sigma\,(kY_1 + \cdots + kY_N) + e^{-Y_1} + \cdots + e^{-Y_N}$$

Integrate out Σ:

$$k(Y_1 + \cdots + Y_N) = 0$$

Since the Y_i are periodic, this is <u>not</u> quite same as, $Y_1 + \cdots + Y_N = 0$.

Rather:

$$e^{-Y_N} = \Upsilon\, e^{Y_1} e^{Y_2} \cdots e^{Y_{N-1}} \quad \text{for } \Upsilon \text{ an undetermined } k^{th} \text{ root of unity}$$

Toda dual theory:

$$W = e^{-Y_1} + \cdots + e^{-Y_{N-1}} + \Upsilon\left(e^{Y_1} e^{Y_2} \cdots e^{Y_{N-1}}\right)$$

- can check B model corr'f'ns here = A model corr'f'ns of orig.
- looks like a deformation by a triv'ly acting $\mathbb{Z}_k$ twist field

Using similar methods,
the LG point mirror of the quintic in $\mathbb{P}^4$ w/ fields of charge k is a $(\mathbb{Z}_5)^4$ orbifold of LG w/

$$W = x_1^5 + \cdots + x_5^5 + \underline{Y}\, x_1 x_2 x_3 x_4 x_5$$

where $\underline{Y}$ is a field valued in k^{th} roots of unity.

So in mirrors to noneffective gaugings,
fields valued in roots of unity are common.

There's a more systematic way to understand these noneffective gauging phenomena:

string compactifications on <u>stacks</u>

- Stacks generalize spaces.

- Every stack has a presentation of the form $[X/G]$,
  $\longleftrightarrow$ G-gauged sigma model on X

  (G need not be finite, need not act effectively.)

Such presentations <u>not</u> unique, so...

<u>Conjecture</u>: universality classes of worldsheet RG flow ~~of~~ gauged sigma models

$\xleftrightarrow{1\text{-}1}$ stacks

<u>Questions</u>: e.g. massless ~~spectra~~ - what is it for G nonfinite, & is it presentation-independent?

<u>Issues</u>: e.g. deformation theory

<u>Uses</u>: Concrete realization of local orbifolds, new LG models, ...

Back to mirror symmetry.

There is a notion of toric stacks.

Data: fan + some abelian finite group data decorating each edge

Recall Batyrev's mirror conjecture:
exchanges polytope for fan w/ Newton polytope of mirror hypersurface

To make Batyrev's conjecture work for stacks, need a way to decorate Newton polygon w/ finite group data

→ but we've already seen how to do this:
e.g. $\underline{\mathcal{V}}\ \pi x_i$: terms in LG superpotential

Conclusions

→ gauging noneffective group actions
$\neq$ gauging effective group actions

→ understanding deformation theory & mirror symmetry leads to a new class of LG models, w/ fields valued in roots of unity

→ part of a larger program of understanding string compactifications on stacks